International Journal of Theoretical Physics, Vol. 43, No. 3, March 2004 (© 2004)

# Some Properties of Multigravity Theories and Discretized Brane Worlds

**C. Deffayet[1,3,4] and J. Mourad[2,3]**

We review some properties of solutions to 5D Einstein gravity with a discrete fifth dimension. Those properties depend on the discretization scheme we use. In particular, we find that the neglect of the lapse field (along the discretized direction) gives rise to Randall–Sundrum-type metric with a negative tension brane. However, no brane source is required. The inclusion of the lapse field gives rise to solutions whose continuum limit is gauge fixed by the discretization scheme. We show also that the models allow a continuous mass spectrum for the gravitons with an effective 4D interaction at small scales.

**KEY WORDS:** multigravity; deconstruction of gravity; massive gravity.

## 1. INTRODUCTION

Theories with many metrics coupled together were investigated in the past in relation to strong interaction (Chamesddine, 2003; Isham *et al.*, 1971; Isham and Storey, 1978; Salam and Strathdee, 1977), and more recently by various authors (Arkani-Hamed *et al.*, 2003; Arkani-Hamed and Schwartz, 2003; Damour *et al.*, 2002, 2003; Damour and Kogan, 2002; Schwartz, 2003) in particular in relation with discrete extra-dimensions (Arkani-Hamed *et al.*, 2001, 2003; Arkani-Hamed and Schwartz, 2003; Hill *et al.*, 2001; Schwartz, 2003). Such theories can be a tool to investigate the various problems associated with "massive" gravity but have also their own interests. In the perspective of better understanding theories with multiple gravitons, it is interesting to contrast such theories obtained from a parent theory which is known to be fully consistent with the parent theory itself. This has

[1] GReCO/IAP (FRE2435 du CNRS) 98 bis boulevard Arago, 75014 Paris, France.
[2] Laboratoire du Physique Théorique (UMR8627 du CNRS), Bât. 210. Université Paris XI, 91405 Orsay Cedex, France.
[3] Fédération de recherche APC, Université Paris VII, 2 place Jussieu - 75251 Paris Cedex 05, France.
[4] To whom correspondence should be addressed at GReCO/IAP 98 bis boulevard Arago, 75014 Paris, France; e-mail: deffayet@iap.fr or mourad@th.u-psud.fr.

been done in the past using Kaluza–Klein compactification (Aulakh and Sahdev, 1985; Dolan and Duff, 1984; Nappi and Witten, 1989).

In the first part of this talk, after a short introduction of the formalism used (Section 2) and developed in Deffayet and Mourad (2003), we follow a similar path (Section 3) by comparing simple solutions of theories obtained by discretizing one space-like dimension in five-dimensional (5D) general relativity to solution of the continuum theory (Deffayet and Mourad, 2004). Namely we consider the extra dimension to be given by a one-dimensional lattice, to each point of the lattice is associated a four-dimensional (4D) space-time with a metric. We determine the coupling of the metrics by the requirement that the continuum limit should be given by five-dimensional gravity. The discrete action should respect as much as possible the symmetries of the 5D action, otherwise new degrees of freedom appear which are in general ghost-like. The actions we shall consider break explicitly the reparametrization invariance along the fifth direction. Although at the linear level the theory is free from ghosts, they may appear at the nonlinear level (Boulware and Deser, 1972).

In the last part of this talk, we show that the theory considered have also the interesting property to yield a 4D gravitational potential at small distances which becomes 5D at large distances, while gravity is mediated by a continuum of massive gravitons (Deffayet and Mourad, 2004). This is similar to what is happening in the brane-induced gravity model of Dvali *et al.* (2000 a).

## 2. DISCRETE VS. CONTINUUM THEORY

For the purpose of discretizing a space-like dimension, parametrized by coordinate $y$, it is convenient to use a $4+1$ splitting of space-time and rewrite (after an integration by part) the 5D Einstein Hilbert action as

$$M_{(5)}^3 \int d^4x\, dy \sqrt{-g}\mathcal{N}\{R - 2\tilde{\Lambda} + K_{\mu\nu}K_{\alpha\beta}(g^{\mu\nu}g^{\alpha\beta} - g^{\mu\alpha}g^{\nu\beta})\}, \qquad (1)$$

where $M_{(5)}$ is the 5D reduced Planck mass, $K_{\mu\nu}$ is the extrinsic curvature of surfaces $\mathcal{H}_y$ located at constant $y$, and we have introduced in a standard way[5] the *lapse* $\mathcal{N}$, the *shift* $N_\mu$, and induced metric $g_{\mu\nu}$ on $\mathcal{H}_y$, whose Ricci scalar is denoted by $R$. The extrinsic curvature is defined by

$$K_{\mu\nu} = \frac{1}{2\mathcal{N}}(g'_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu), \qquad (2)$$

where $D_\mu$ is the covariant derivative associated with the induced metric $g_{\mu\nu}$ and a prime denotes an ordinary derivative with respect to $y$. The fields $\mathcal{N}$, $N_\mu$, and $g_{\mu\nu}$ are simply related to the components of the 5D metric $\tilde{g}_{AB}$ by

$$\tilde{g}_{\mu\nu} = g_{\mu\nu}, \qquad (3)$$

[5] Note however that the surface located at constant $y$ are time-like, unlike in the usual ADM splitting.

$$\tilde{g}_{\mu y} = N_\mu \equiv g_{\mu\alpha} N^\alpha, \tag{4}$$

$$\tilde{g}_{yy} = \mathcal{N}^2 + g_{\mu\nu} N^\mu N^\nu. \tag{5}$$

Theories which will be of interest in this work can be obtained from the Einstein–Hilbert action (1), where one discretizes the continuous coordinate $y$ with a spacing $a$ between two adjacent sites (labelled by an index $i$). It was shown in Deffayet and Mourad (2003) how to obtain such a discretization, maintaining $y$-dependent 4D gauge invariance on each site. This can be done by the mean of *link fields* $X^\mu$ $(i, i+1; x)$ (Arkani-Hamed *et al.*, 2003; Arkani-Hamed and Schwartz, 2003; Schwartz, 2003), mapping between site $i$ and site $i+1$, which were explicitly built out of the $5D$ metric in Deffayet and Mourad (2003). In analogy with transverse latticification of gauge theories the link field buildup in Deffayet and Mourad (2003) is a path-ordered exponential the vector field $N$ acting on coordinate functions $x^\mu$. At leading order in the transverse lattice spacing $a$, $X^\mu$ is given by

$$X^\mu(i, i+1, x) = x^\mu + aN^\mu(y_i, x) + \mathcal{O}(a^2), \tag{6}$$

where $y_i$ is the $y$ coordinate of site $i$ (one has $y_i \equiv ia$). We then replace the $y$ derivatives, appearing in action (1) only in the extrinsic curvature $K_{\mu\nu}$, by expressions made up from a transport operator built out of the link field and acting on the metric of each site (see Deffayet and Mourad, 2003, 2004. If one then makes a gauge choice such that $\mathrm{X}^\mu(i, i+1, x) = x^\mu$, one is led to consider theories of a set of 4D metrics $g^i_{\mu\nu}$, and 4D scalar lapse field $\mathcal{N}_i$ with actions of the form

$$S[g_i, \mathcal{N}_i] = \Sigma_i M^2_{(4)} \int d^4x \sqrt{-g_i} \mathcal{N}_i (R(g_i) - 2\Lambda) - \int d^4x \frac{M^2_{(4)}}{\mathcal{N}_i} V(g_i, g_{i+1}), \tag{7}$$

where $V(g_i, g_{i+1})$ is an interaction term between the metrics $g^i_{\mu\nu}$ and $g^{i+1}_{\mu\nu}$, and $M_{(4)}$ is a mass scale which sets the coupling scale between the metric on a given site and matter sources that one may wish to put on the same site.

Action (7) is a simple- minded discretization of the 5D pure gravity Einstein-Hilbert action (1), as can be seen explicitly using the following identification:

$$m^2 = \frac{1}{a^2}, \tag{8}$$

$$M^2_{(4)} = M^3_{(5)} a, \tag{9}$$

$$\Lambda = \tilde{\Lambda}, \tag{10}$$

$$g^i_{\mu\nu}(x^\mu) = \tilde{g}_{\mu\nu}(x^\mu, y_i), \tag{11}$$

$$\mathcal{N}_i(x^\mu) = \mathcal{N}(x^\mu, y_i). \tag{12}$$

If one insists in keeping the link with the 5D theory, one should verify the equation of motion for $X^\sigma$. The latter read (in the gauge $X^\mu = x^\mu$)

$$0 = \frac{2\mathcal{N}_i}{\sqrt{-g_i}}\partial_\mu\left(\frac{\sqrt{-g_i}}{\mathcal{N}_i}g^{i+1}_{\sigma\nu}\left(g^{i+1}_{\alpha\beta} - g^{i}_{\alpha\beta}\right)\left(g_i^{\mu\nu}g_i^{\alpha\beta} - g_i^{\mu\alpha}g_i^{\nu\beta}\right)\right)$$
$$-\left(\partial_\sigma g^{i+1}_{\mu\nu}\right)\left(g^{i+1}_{\alpha\beta} - g^{i}_{\alpha\beta}\right)\left(g_i^{\mu\nu}g_i^{\alpha\beta} - g_i^{\mu\alpha}g_i^{\nu\beta}\right), \tag{13}$$

where $g_i^{\mu\nu}$ is the inverse metric of $g^i_{\mu\nu}$. This reduces to the equation of motion of the shift in the continuum limit. This equation is somehow similar to a Kaluza-Klein consistency condition (Duff *et al.*, 1984; Jordan, 1947; Thiry, 1948). Note that the index $i$ can be envisioned as labelling *theory space* sites in the spirit of the deconstruction program of the literature (Arkani-Hamed *et al.*, 2003; Arkani-Hamed and Schwartz, 2003; Schwartz, 2003), but theories under investigation here can also be considered without an explicit reference to a continuum limit, simply as theories of *multigravity* (Chamesddine, 2003; Isham *et al.*, 1971; Isham and Storey, 1978; Damour *et al.*, 2002, 2003; Damour and Kogan, 2002; Salam and Strathdee, 1977). In the latter case, one does not have to consider Eq. (13). We will, for most of the cases discussed in this paper, not include any matter fields so that each of the sites $i$ will only be considered endowed with a cosmological constant $\Lambda$.

## 3. DISCRETIZED BRANE WORLDS

We first set from the beginning the lapse fields $\mathcal{N}_i$ to one in the multigravity action $S[g_i, N_i]$ and consider solutions to the equations of motion derived from the simplified action for the metrics $g^i_{\mu\nu}$, $S[g_i, 1]$. We wish here to compare these solutions with solutions of the continuum theory defined by action (1) and seek solutions of the form

$$g^i_{\mu\nu} = \Omega^i\eta_{\mu\nu} \tag{14}$$

with $\Omega^i$ constants. With such a choice, Eq. (13) is automatically fulfilled. After a straightforward calculation, the equation of motion for the metric $g_i$ reduces to the sequence defined by

$$-\lambda = -2 + f_i + (f_{i-1})^{-1}, \tag{15}$$

where $\lambda = 2\Lambda/3\mathrm{m}^2$ and $f_i = \Omega_{i+1}/\Omega_i$. The general solution of which is given by

$$\Omega^j = ((f_+)^j + \mathcal{K}(f_-)^j)\frac{\Omega^0}{1+\mathcal{K}}, \tag{16}$$

where $f_+$ and $f_-$ are the two fixed points (for $\lambda$ obeying $\lambda(-1+\lambda/4) > 0$) of the sequence (15), and $\mathcal{K}$ is an integration constant, related to $f_0$ by

$$\mathcal{K} = \frac{f_+ - f_0}{f_- - f_0}. \tag{17}$$

If one chooses $f_i$ to lie initially on one of the fixed points, the solution for the conformal factor $\Omega^i$ is given by

$$\Omega^i_{\pm} = \Omega^0_{\pm}(f_{\pm})^{y_j/a}. \tag{18}$$

In the continuum limit when the the discretization step a goes to zero, this leads to

$$g^j_{\mu\nu} \sim \Omega^0_{\pm} \exp\left(\pm\sqrt{-\frac{2}{3}\tilde{\Lambda}}y_j\right)\eta_{\mu\nu}. \tag{19}$$

It matches a Poincaré patch of $AdS_5$ parametrized à la Randall–Sundrum (namely it is the metric used in Randall and Sundrum (1999a,b) if one removes there the absolute value in the exponential, that is to say without cutting off the AdS boundary by a positive tension brane). On the other hand, the most general solution (for a positive $\mathcal{K}$) has a continuum limit given by

$$g^i_{\mu\nu} = \frac{2\Omega^0\sqrt{\mathcal{K}}}{(1+\mathcal{K})} \cosh\left(\sqrt{-\frac{2}{3}\Lambda(y_i-\gamma)}\right)\eta_{\mu\nu}, \tag{20}$$

where $\gamma$ is a constant given by $\gamma = \sqrt{-3/8\Lambda}\ln(\mathcal{K})$. This solution has exactly the same asymptotics as $y_i$ goes to $\pm\infty$ as the Randall-Sundrum metric (Randall and Sendrum, 1999a,b) with a negative tension brane placed at $y = \gamma$. This is a quite remarkable feature since no brane has been considered. One can show that this result is robust under a change in the disretization procedure considered (Deffayet and Mourad, 2004).

There is an easy way to understand these solutions by comparing the equations of motion of the continuum theory (defined by action (1)) to the ones of the multigravity theory (15). If one seeks a solution of the continuum theory of the form $g_{\mu\nu}(x^\alpha, y) = \Omega(y)\eta_{\mu\nu}$, with $\mathcal{N}$ set to one; one finds that the most general solution of the equation of motion for $g_{\mu\nu}$ deduced from action (1) is given for $\Omega$ by a linear combination of exp $(ky)$ and exp $(-ky)$ with $k$ given by $\sqrt{-2\Lambda/3}$. This matches with what is found in the discretized theory. Indeed, in the limit where $a$ goes to zero, Eq. (15) reduces to equation the equation of motion for the metric $g_{\mu\nu}$ deduced from action (1). However, in the continuum theory, the equation of motion for $\mathcal{N}$ allows to keep the decreasing or increasing exponential, but not a combination of the two. This also enables to understand that we did not find the solution with a positive tension brane (which would have been very interesting in many respects). Indeed, a linear combination of exponentials exp $\pm ky$ (with positive coefficients) is an increasing function for large positive $y$ and a decreasing function for large negative $y$. So that the asymptotic *jump* of the first $y$ derivative of the 4D metric across the *brane*, defined as $[\Omega'(+\infty) - \Omega'(-\infty)]/[\Omega(+\infty) + \Omega(-\infty)]$, is necessarily positive. This in turn means that the brane tension has to be negative, as can be seen from the junction conditions.

It is possible to find also in the discrete theory considered so far a discretized Randall-Sundrum space-time with a positive tension brane. To do so, one can introduce a localized brane source in the discrete theory. This amounts to change in the cosmological constant $\Lambda$ of action (7) at one site. The value of this cosmological constant must then be precisely tuned (in a manner shown in Deffayet and Mourad, 2004) to the value of the cosmological constant on the other sites, in a way similar to what is happening in the continuum theory. In the continuum limit, this tuning gives back the Randall-Sundrum tuning condition between the brane tension and the bulk cosmological constant (Randall and Sundrum, 1999 a,b).

So far we have only discussed cases where the $\mathcal{N}_i$ fields were dropped from the action (7). However, as we just discussed, this dropping introduces spurious solutions when compared with solutions of the continuum theory. If one then reintroduces $\mathcal{N}_i$ and seeks solutions similar to the previous, one can as well readily derive equations of motions from action (7), which read

$$\sqrt{-\lambda \mathcal{N}_i} = \epsilon_i (f_i - 1), \tag{21}$$

$$-\lambda \mathcal{N}_i = \frac{1}{\mathcal{N}_i}(f_i - 1) - \frac{1}{\mathcal{N}_{i-1}}\left(1 - \frac{1}{f_{i-1}}\right), \tag{22}$$

here $\epsilon_i = \pm 1$. This can be solved for both $\mathcal{N}_i$ and $\Omega_i$. The obtained solution can be seen to be, as previously, a discretized given parametrization of a Poincaré patch of $AdS_5$ (where the gauge is set by the disretization scheme chosen). The fact that one can solve for both $\mathcal{N}_i$ and $\Omega_i$ is a consequence of the fact that the $y$-reparametrization invariance is explicitly broken in the discrete theory. One can however choose a discretization scheme which restores the indetermination of $\mathcal{N}_i$, $e.g.$, by choosing a potential given by

$$V(g_{i-1}, g_i, g_{i+1}) = -\frac{m^2}{16}\sqrt{-g_i}\left(g^{i+1}_{\mu\nu} - g^{i-1}_{\mu\nu}\right)\left(g^{i+1}_{\alpha\beta} - g^{i-1}_{\alpha\beta}\right)\left(g_i^{\mu\nu} g_i^{\alpha\beta} - g_i^{\mu\alpha} g_i^{\nu\beta}\right). \tag{23}$$

With this interaction term, the action $S[g_i, \mathcal{N}_i]$ also agrees with action (1) in the continuum limit. However doing so, one finds again some solutions which do not have any continuum counterpart, namely change in the signature from site to site.

## 4. EFFECTIVE 4D GRAVITY

We turn now to determine the gravitational potential without the lapse field (i.e., in the theory $S[g_i, 1]$) and consider the discretization scheme given in the previous subsection. We assume here that the cosmological constant vanishes and so we can perturb around flat space-time. To quadratic order in the metric perturbation $h^i_{\mu\nu}$, the interaction term (23) reads

$$V(h_{i-1}, h_i, h_{i+1}) = -\frac{m^2}{16}\left(h^{i+1}_{\mu\nu} - h^{i-1}_{\mu\nu}\right)\left(h^{i+1}_{\alpha\beta} - h^{i-1}_{\alpha\beta}\right)(\eta^{\mu\nu}\eta^{\alpha\beta} - \eta^{\mu\alpha}\eta^{\nu\beta}). \tag{24}$$

To diagonalize this interaction, we define $\check{h}_{\mu\nu}$ by

$$\check{h}_{\mu\nu}(x^\alpha,\theta)=\sum_n h^n_{\mu\nu}(x^\alpha)\,e^{in\theta}. \tag{25}$$

The quadratic action is then an integral over $\theta$ of Pauli–Fierz actions with a continuous mass spectrum given by

$$m^2(\theta)=\frac{\sin^2\theta}{a^2}. \tag{26}$$

And the gravitational potential $\Psi_j(r)$, between two unit masses separated by $r$ and placed at sites $i$ and $i+j$, can be obtained summing over Pauli-Fierz propagators. The outcome of the calculation can also be simply understood from discretizing the Laplacian equation with the same discretization scheme as above. The discretized equation reads

$$\partial^2\Psi_j+\frac{(\Psi_{j+2}+\Psi_{j-2}-2\Psi_j)}{4a^2}=4\pi G_{\mathrm{N}}\delta(\mathbf{r})\delta_{\mathbf{j,o}}, \tag{27}$$

where we have reintroduced the Newton constant $G_{\mathrm{N}}$. The Fourier transform $\check{\Psi}$, as defined above, verifies

$$\partial^2\check{\Psi}(r,\theta)-m^2(\theta)\check{\Psi}(r,\theta)=4\pi G_{\mathrm{N}}\delta(\mathbf{r}). \tag{28}$$

Notice that the mass spectrum is bounded from above by the inverse lattice spacing $a^{-1}$. A continuous mass spectrum is reminiscent of the infinite dimensional models of Dvali *et al.* (2000b) and Gregory *et al.* (2000). The gravitational potential can now be readily put in the form

$$\Psi_j(r)=-\frac{G_{\mathrm{N}}}{r}=\int_0^\pi\frac{d\theta}{\pi}\,e^{-\frac{r}{a}\sin\theta}\cos j\theta \tag{29}$$

When $r\ll a$ then the integral can be approximated by $\delta_{j,0}$ and the potential reduces to the 4D Newtonian potential

$$\Psi_j(r)=-\frac{G_{\mathrm{N}}}{r}\left(\delta_{j,0}+O\left(\frac{r}{a}\right)\right),\quad r\ll a. \tag{30}$$

When $r\gg a$, then the integral can be approached by

$$\int_0^{\pi/2}\frac{d\theta}{\pi}\,e^{-\frac{r}{a}\theta}\cos j\theta+\int_{\pi/2}^{\pi}\frac{d\theta}{\pi}\,e^{-\frac{r}{a}(\pi-\theta)}\cos j\theta$$
$$=\frac{1}{\pi a}\frac{r}{r^2+a^2j^2}(1+(-1)^j+O(e^{\frac{-r}{a}})), \tag{31}$$

so that the gravitational potential is of the form of a 5D potential

$$\Psi_j(r)=-\frac{G_5}{r^2+(ja)^2}, \tag{32}$$

if $j$ is even, with the 5D gravitational constant being given by

$$G_5 = \frac{2G_{\rm N}}{\pi a}. \tag{33}$$

When $j$ is odd, the gravitational potential is exponentially small. Gravity is thus four dimensional at small length scales and five dimensional at large scales. Choosing $a$ very large (of the order of the Hubble scale) allows a very simple modification of gravity at large scales in the spirit of the models of references (Dvali *et al.*, 2000b; Gregory *et al.*, 2000; Kogan *et al.*, 2000; Kogan and Ross, 2000). Note that we could have started from a finite number $N$ of sites. This corresponds to an extra dimension which is compact with a length scale $R = Na$. The graviton spectrum would have been discrete with a spacing of order $1/N$ and would have remained bounded. In this case, the 4D regime is obtained for both small scales $r \ll a$ and large scales $r \gg R$, the intermediate scales being five dimensional $a \ll r \ll R$. The qualitative features of this potential are not sensitive to the particular form of the discretization scheme we have used. On the other hand, it is well known that the tensorial structure of the propagator of massive spin 2 fields differs dramatically from the massless one. This leads to the vDVZ discontinuity at the linearized level which is manifested by, e.g., an order 1 difference in light bending (Iwasaki, 1970; van Dam and Veltman, 1970; Zakharov, 1970). In this respect, when $R$ is infinite, we expect the discontinuity to be present since we have a continuous spectrum of massive gravitons. This is similar to the brane models with an extra infinite dimension (Dvali *et al.*, 2000b; Gregory *et al.*, 2000; Kogan *et al.*, 2000; Kogan and Ross, 2000). When $R$ is finite, however, the spectrum is discrete and there is no discontinuity.

## 5. CONCLUSIONS

In this talk, we have summarized some properties of solutions of 5D general relativity with a discrete fifth dimension and compared them to solutions of the continuum theory. Those solutions were discussed more extensively in Deffayet and Mourad (2004) with some other solutions (in particular cosmological solutions). Depending on the discretization scheme used, we have shown that some of the solutions of the discrete theory exactly match those of the continuum, while others do not. In general, the discretization explicitly breaks reparametrization along the discrete dimension. This gets reflected in the fact that for solution we have considered, corresponding to slicing of $AdS_5$ by Minkowski space, the equations of motion of the discrete theory enable in general to determine both the lapse field and the 4D metric. This is to be contrasted with the continuum theory where the lapse field cannot be determined by the equations of motion. However, we have shown that one can find a given discretization scheme in which it remains undetermined. More importantly, some of the solutions of the discrete theory exhibit very dramatic differences with those of the continuum. As shown in Deffayet and Mourad (2004), one can find signature change of the 4D metric but also avoidance of singularities which would be present in the continuum. In addition we have also found a brane world looking bulk space-time, with no brane source in the equation of motion and no fine-tuning. This solution would correspond in the continuum theory to a negative tension brane. We have investigated the static gravitational potential and found it realistic when the lattice spacing is very large.

We can look at our results from two different perspectives. On one hand, they exemplify the difficulties which arise upon deconstruction of gravity even at the classical level: the neglect of the lapse fields leads to spurious solutions, while their inclusion only partially solves this problem, On the other hand, they point out interesting directions in multigravity theories: they allow, as we showed, simple modifications of gravity at large scales and brane-like solutions with no branes. One can speculate on the possibility to reproduce these results, in some more complex multigravity, without the various drawbacks mentioned in the Introduction. It would also be interesting to find a solution corresponding to the discretization of RS type—gravity localizing—metric (Randall and Sundrum, 1999a,b), and at the same time avoiding fine-tuning conditions.

## ACKNOWLEDGMENTS

We thank E. Dudas and S. Pokorski for helpful discussions. C.D. is very grateful to the organizers of the 8th Peyresq Physics meeting on “The early univers: confronting theory with observations” for their invitation. He especially thanks Diane and Edgard Gunzig for their warm welcome and organization.

## REFERENCES

Arkani-Hamed, N., Cohen A. G., and Georgi, H. (2001). (De)constructing dimensions, *Physical Review Letters* **86**, 4757 [arXiv:hep-th/0104005].

Arkani-Hamed, N., Georgi H., and Schwartz, M. D. (2003). Effective field theory for massive gravitons and gravity in theory space, *Annals of Physics* **305**, 96 [arXiv:hep-th/0210184].

Arkani-Hamed N. and Schwartz, M. D., (2003) Discrete gravitational dimensions, arXiv:hep-th/0302110;

Aulakh C. S. and Sahdev, D. (1985). The infinite dimensional gauge structure of KaluzaKlein theories. 1. D = 1+4, *Physics Letters B* **164**, 293.

Boulware D. G. and Deser, S. (1972). Can gravitaton have a finite range?, *Physical Review D: Particles and Fields* **6**, 3368.

Chamseddine, A. H. (2003). Spontaneous symmetry breaking for massive spin-2 interacting with gravity. *Physics Letters B* **557**, 247 [arXiv:hep-th/0301014].

Damour T. and Kogan, I. I. (2002). Effective lagrangians and universality classes of nonlinear bigravity. *Physical Review D: Particles and Fields* **66**, 104024 [arXiv:hep-th/0206042].

Damour, T., Kogan I. I., and Papazoglou, A. (2002). Non-linear bigravity and cosmic acceleration. *Physical Review D: Particles and Fields* **66**, 104025 [arXiv:hep-th/0206044].

Damour, T., Kogan I. I., and Papazoglou, A. (2003). Spherically symmetric spacetimes in massive gravity. *Physical Review D: Particles and Fields* **67**, 064009 [arXiv:hep-th/0212155].

Deffayet, C., Dvali, G. R., Gabadadze G., and Vainshtein, A. I. (2002). Nonperturbative continuity in graviton mass versus perturbative discontinuity. *Physical Review D: Particles and Fields* **65**, 044026 [arXiv:hep-th/0106001].

Deffayet, C. and Mourad, J. (2003). Multigravity from a discrete extra dimension. *Physics Letters B* **589**, 48 [arXiv:hep-th/0311124].

Deffayet C. and Mourad, J. (2004). Solutions of multigravity theories and discretized brane worlds. *Classical and Quantum Gravity* **21**, 1833, [arXiv:hep-th/0311125].

Dolan L. and Duff, M. J. (1984). Kac-moody symmetries of Kaluza-Klein theories, *Physical Review Letters* **52**, 14.

Dvali, G. R., Gabadadze G., and Porrati, M. (2000a). 4D gravity on a brane in 5D Minkowski space. *Physics Letters B* **485**, 208 [arXiv:hep-th/0005016].

Dvali, G. R., Gabadadze G., and Porrati, M. (2000b). Metastable gravitons and infinite volume extra dimensions. *Physics Letters B* **484**, 112 [arXiv:hep-th/0002190].

Duff, M. J., Nilsson, B. E., Pope C. N., and Warner, N. P. (1984). On the consistency of the Kaluza-Klein Ansatz. *Physics Letters B* **149**, 90.

Gregory, R., Rubakov, V. A., and Sibiryakov, S. M. (2000). Opening up extra dimensions at ultra-large scales. *Physical Review Letters* **84**, 5928 [arXiv:hep-th/0002072].

Hill, C. T., Pokorski S., and Wang, J. (2001). Gauge invariant effective lagrangian for Kaluza-Klein modes. *Physical Review D: Particles and Fields* **64**, 105005 [arXiv:hep-th/0104035].

Isham, C. J., Salam, A., and Strathdee, J. (1971). F-Dominance of gravity. *Physical Review D: Particles and Fields* **3**, 867.

Isham, C. J. and Storey, D. (1978). Exact spherically symmetric classical solutions for the F-G theory of gravity. *Physical Review D* **18**, 1047.

Iwasaki, Y. (1970). Consistency condition for propagators, *Physical Review D: Particles and Fields* **2**, 2255.

Jordan, P. (1947). Erweitung der projektiven relativitätstheories. *Annalen der Physik* (Leipzig) **VI**(1), 219–228.

Kogan, I. I., Mouslopoulos, S., Papazoglou, A., Ross G. G., and Santiago, J. (2000). A three three-brane universe: New phenomenology for the new millennium?. *Nuclear Physics B* **584**, 313 [arXiv:hep-ph/9912552].

Kogan I. I. and Ross, G. G. (2000). Brane universe and multigravity: Modification of gravity at large and small distances. *Physics Letters B* **485**, 255 [arXiv:hep-th/0003074].

Luty, M. A., Porrati M., and Rattazzi, R. (2003). Strong interactions and stability in the DGP model. arXiv:hep-th/0303116.

Nappi C. R. and Witten, L. (1989). Interacting lagrangian for massive spin two field. *Physical Review D: Particles and Fields* **40**, 1095.

Randall L. and Sundrum, R. (1999a). A large mass hierarchy from a small extra dimension. *Physical Review Letters* **83**, 3370 [arXiv:hep-th/9905221].

Randall L. and Sundrum, R. (1999b). An alternative to compactification. *Physical Review Letters* **83**, 4690 [arXiv:hep-th/9906064].

Rubakov, V. A., Strong coupling in brane-induced gravity in five dimensions, arXiv:hep-th/0303125.

Salam, A. and Strathdee, J. (1977). A class of solutions for the strong gravity equations. *Physical Review D* **16**, 2668.

Schwartz, M. D. (2003). Constructing gravitational dimensions, arXiv:hep-th/0303114.

Thiry, Y. (1948). Les équations de la théorie unitaire de Kaluza. *CRAS*, 216.

Vainshtein, A. I. (1972). To the problem of nonvanishing gravitation mass. *Physics Letters B* **39**, 393.

Van Dam H. and Veltman, M. J. (1970). Massive and massless Yang-Mills and gravitational fields. *Nuclear Physics B* **22**, 397.

Zakharov, V. I. (1970). *JETP Letters* **12**, 312.